Bernoulli distribution

the Bernoulli distribution is a model for an experiment that has only two possible outcomes. When a random variable must assume one of the two values, 0 or 1 such a variable is called a Bernoulli random variable. The corresponding experiment, which has only two possible is said to be Bernoulli trial.

There are three assumptions of Bernoulli trials.

1. Each trial has two possible outcomes called success and failure.
2. The trials are independent, the outcome of one trial has no influence over the outcome of another trial.
3. The probability of success in each trial denoted as p is remains constant from trial to trial where $0 < p < 1$.

Ex: sent a message through a network and record whether or not it is received

A= {successful transmission, unsuccessful transmission}

The p.m.f of Bernoulli distribution can write as

$$p(x) = p^x(1-p)^{1-x} \qquad ; x = 0,1$$

$$X \sim b(1, p)$$

- For a Bernoulli distribution, $\mu_x = p$. can easily derive this from the general equation for mean of a discrete random variable:

$$\mu_x = \sum_{i=0}^{1} x_i \, Pr(X = x)$$

$$= \sum_{i=0}^{1} x_i p^x (1-p)^{1-x}$$

$$\mu_x = 0(p)^0 * 1(1-p)^{1-0} + 1 * p^1 * (1-p)^{1-1} = p$$

$$v(x) = E(x-\mu)^2 = \sum_{i=0}^{1} (x_i - \mu)^2 p(x_i)$$

$$= (0-p)^2 * p^0(1-p)^1 + (1-p)^2 * p^1(1-p)^0$$

$$= p^2(1-p) + p(1-p)^2$$

$$= p(1-p)[p+1-p] = p(1-p) = v(x)$$

- $momen\ generating\ gunction\ is$

$$M_x(t) = 1 - p + pe^t$$

Binomial distribution

The probability distribution of a random variable X representing the number of successes in a sequence of n Bernoulli trials, regardless of the order in which they occur is frequently of considerable .it is clearly that X is a discrete r.v. assuming value 0,1, 2, …, n.

An experiment has a binomial probability distribution if three conditions satisfied

1. There are fixed number of trials.
2. The trials are independent.
3. The only outcomes of this experiment can be classified as "succeed " or "fail", the probability of success is fixed and denoted by p.

$$X \sim b(n, p)$$

The probability function for a binomial random variable is

$$f(x) = b(x; n, p) = \binom{n}{x} p^x (1-p)^{n-x} \quad ;\ x = 0,1, \dots, n$$

This is the probability of having x successes in a series of n independent trials when the probability of success in any one of the trials is p

The mean and variance can be found as

$$E(X) = \sum_{x=0}^{n} x \binom{n}{x} p^x (1-p)^{n-x}$$

$$= \sum_{x=0}^{n} x \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}$$

$$= \sum_{x=1}^{n} \frac{n!}{(x-1)!\,(n-x)!} p^x (1-p)^{n-x}$$

since the x = 0 term vanishes. Let y = x−1 and m = n−1. Subbing x = y+1 and n = m+1 into the last sum (and using the fact that the limits x = 1 and x = n correspond to y = 0 and y = n − 1 = m, respectively

$$E(X) \quad = \textstyle\sum_{y=0}^{m} \frac{(m+1)!}{y!\,(m-y)!} p^{y+1} (1-p)^{m-y}$$

$$= (m+1)p \textstyle\sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^{y} (1-p)^{m-y}$$

$$= np \textstyle\sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^{y} (1-p)^{m-y}$$

The binomial theorem says that

$$(a+b)^m = \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^y b^{m-y}$$

Setting a = p and b = 1 −p

$$\sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} p^y (1-p)^{m-y} = \sum_{y=0}^{m} \frac{m!}{y!\,(m-y)!} a^y b^{m-y}$$

$$= (a+b)^m = (p+1-p)^m = 1$$

$$\therefore E(x) = np$$

$E(x^2) = n^2p^2 - np^2 + np$

$$v(x) = E(x^2) - \{E(x)\}^2 = n^2p^2 - np^2 + np - (np)^2$$
$$= n^2p^2 - np^2 + np - n^2p^2 = np - np^2 = np(1-p)$$
$$= v(x)$$

Ex: each sample of water has 10% chance of containing a particular organic pollutant find probability that in the next 18 sample exactly 2 contain the pollutant.

Sol\

$$X \sim b(18,0.1)$$

$$f(x) = \binom{n}{x} p^x (1-p)^{n-x} \quad ; \; x = 0,1,\dots,18$$

$$= \binom{18}{2} 0.1^2 (1-0.1)^{18-2} = \binom{18}{2} 0.1^2 (0.9)^{16} = 0.284$$

Ex: team A has prob.=2/3 of wining whenever it plays. If A play 4 games find the prob. That A wins

1. Exactly 2 games.
2. At least one game.
3. More than half of gams.

Sol\

1.

$$p(x=2) = \binom{4}{2}\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 = \frac{24}{81}$$

2. $p(At\ least\ one\ game\ win) = 1 - p(on\ win)$

$$= 1 - \left[\binom{4}{0}\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^4\right] = 1 - \frac{1}{81} = \frac{80}{81}$$

3. $P(more\ than\ half\ games) = p(x=3) + p(x=4)$

$$= \binom{4}{3}\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)^1 + \binom{4}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^0 = \frac{32}{81} + \frac{16}{81} = \frac{48}{81}$$

Poisson Distribution

The Poisson distribution is used to model the number of events occurring within a given time interval.

The formula for the Poisson probability mass function is

$$p(x;\lambda) = \frac{e^{-\lambda}\lambda^{x}}{x!} \text{ for } x = 0,1,2,\cdots$$

- The Poisson random variable has large range of application. a major reason for this is that a Poisson random variable can be used as an approximation for a binomial random variable with parameter (n, p) when n is large and p is small.

λ is the shape parameter which indicates the average number of events in the given time interval.

- $\lambda = \mathrm{E}(X) = \mathrm{Var}\,(X)$