CH6: Expectation of random variable

in probability theory, the expected value of a random variable denoted E$\{x\}$ is a generalization of the weighted average, and is intuitively the arithmetic mean of a large number of independent realizations of X. The expected value is also known as the expectation, mathematical expectation, mean, average, or first moment. Expected value is a key concept in economics, finance, and many other subjects.

Def: the expectation value of same function of $x, g(x)$ which we denoted by $E\{x\}$ *, or* $E\{g(x)\}$ is given by

- if X is discrete *r.v.*with *p.m.f* $p(x_i)$

$$E\{x\} = \sum_{i=1}^{n} x\, p(x_i)$$

and in general

$$E\{g(x)\} = \sum_{i=1}^{n} g(x_i)\, p(x_i)$$

- if X is continuous *r.v.* with *p.d.f* $f(x)$

$$E\{x\} = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

and in general

$$E\{g(x)\} = \int_{-\infty}^{\infty} g(x)\, f(x)\, dx$$

- *in betting games, the expectation represents the return of the game to the player, and the game is called suitable for the player if the expectation is positive. it is not suitable if the expectation is negative. but if the expectation is zero, the game is called fair.*

ex: a player tosses a fair dice if a prime number occurs, he wins that number of dollars but if a non-prime number occurs, he losses that number of dollars, find the expected value

sol\

	non-prime	prime	non-prime	prime	non-prime	prime
x	-1	+2	+3	- 4	+5	-6
$p(x)$	1/6	1/6	1/6	1/6	1/6	1/6

fair dice

$$E\{x\} = \sum_{i=1}^{n} x\, p(x_i) = \sum_{i=1}^{6} x\, p(x_i)$$

$$= -1 * \left(\frac{1}{6}\right) + 2 * \left(\frac{1}{6}\right) + \left(3 * \left(\frac{1}{6}\right)\right) + \left(-4 * \left(\frac{1}{6}\right)\right) + \left(5 * \left(\frac{1}{6}\right)\right) + \left(-6 * \left(\frac{1}{6}\right)\right)$$

$$= \left(\frac{1}{6}\right)\{-1+2+3-4+5-6\}$$

$$= \left(\frac{1}{6}\right)\{-1\} = -\frac{1}{6}$$

unfair game for the player because he lost (negative sign)

ex: find mathematical expectation of *r.v.* of the following p.d.f.

$$f(x) = \begin{cases} \frac{1}{2}x, & 0 < x < 2 \\ 0, & o.w. \end{cases}$$

Sol\

$$E\{x\} = \int_{-\infty}^{\infty} x\, f(x)\, dx$$

$$E\{x\} = \int_{0}^{2} x \frac{1}{2} x\, dx = \frac{1}{2}\int_{0}^{2} x^2\, dx = \frac{1}{2}\left(\frac{1}{3}x^3\Big|_0^2\right) = \frac{1}{6}(2^3 - 0) = \frac{8}{6}$$

<u>Probertites of expectation</u>

1. $E(a) = a \quad ; a \text{ is constant}$
2. $E(ax) = a\, E(x)$
3. $E(a \pm x) = a \pm\ E(x)$
4. $E(x+y) = E(x) + E(y)$
5. $E(xy) \neq E(x)E(y)$

- If X and Y are independent then $E(xy) = E(x)E(y)$

Ex: a fair dice is tossed, let X denoted twice the number appearing. Let Y denoted 1 or 3 according as an odd or an even number appears find

1. $E(x+y)$
2. $E(xy)$

Sol\

x	2	4	6	8	10	12
$p(x)$	1/6	1/6	1/6	1/6	1/6	1/6

X=odd → $y = 1$

y= (1,3,5)→ $p(y) = \frac{3}{6} = \frac{1}{2}$

X=even → $y = 3$

y= (2,4,6)→ $p(y) = \frac{3}{6} = \frac{1}{2}$

y	1	3
$p(y)$	1/2	1/2

$$E\{x\} = \sum_{i=1}^{n} x\, p(x_i)$$

$$= 2\left(\frac{1}{6}\right) + 4\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) + 8\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right) + 12\left(\frac{1}{6}\right) = \frac{42}{6} = 7$$

$$E\{y\} = \sum_{i=1}^{n} y\, p(y_i)$$

$$= 1\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right) = \frac{4}{2} = 2$$

1. $E(x+y) = E(x) + E(y) = 7 + 2 = 9$

x	2	4	6	8	10	12
y	1	3	1	3	1	3
xy	2	12	6	24	10	36
$p(xy)$	1/6	1/6	1/6	1/6	1/6	1/6

$$E(xy) = \sum_{i=1}^{6} x_i y_i \, p(x_i y_i)$$

$$= 2\left(\frac{1}{6}\right) + 12\left(\frac{1}{6}\right) + 6\left(\frac{1}{6}\right) + 24\left(\frac{1}{6}\right) + 10\left(\frac{1}{6}\right) + 36\left(\frac{1}{6}\right)$$

$$E(xy) = \frac{90}{6} = 15$$

H.W (+3)

a sample of 3 item is selected at random from a box containing 12 items of which 3 are defective. find the expected number of defective items.