A Finite Stochastic Process and Tree Diagram

each experiment has a finite number of outcomes with given probability is called finite stochastic process a convenient way of describe such process and computing the probability of event is by a tree diagram

Ex: we are given three boxes as follows:

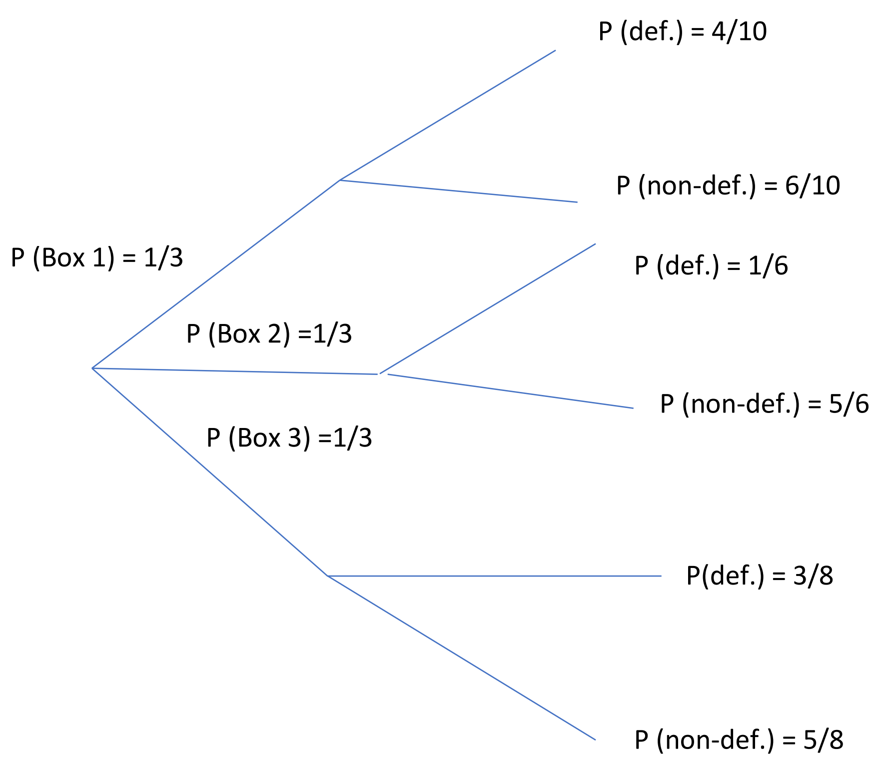
Box 1 has 10 lights bulb of which 4 defectives.

Box 2 has 6 lights bulb of which 1 defective.

Box 3 has 8 lights bulb of which 3 defectives.

we draw a bulb at random what is the probability that bulb is defective

sol\

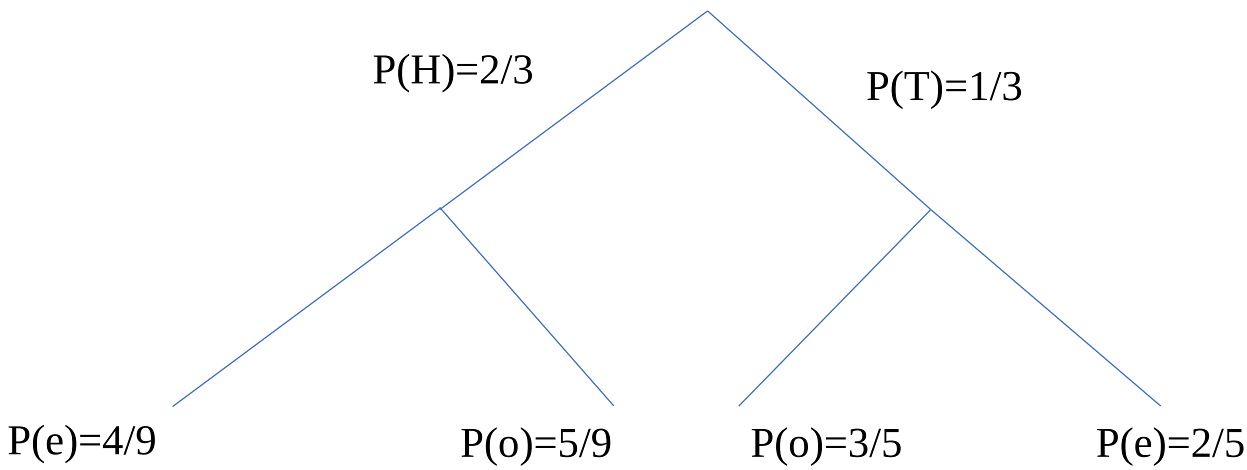


$$p(the\ bulb\ is\ defectiv) = \frac{1}{3} * \frac{4}{10} + \frac{1}{3} * \frac{1}{6} + \frac{1}{3} * \frac{3}{8} = \frac{113}{360}$$

Ex: a coin weighted so that $p(H) = \frac{2}{3}$, $p(T) = \frac{1}{3}$ is tossed if head appears then the number is selected at random from the numbers 1 through 9; if tail appears then the numbers is selected at random from 1 to 5 find prob. That odd number is selected

Sol\

e=even, o= odd



$$p(odd) = \frac{2}{3} * \frac{5}{9} + \frac{1}{3} * \frac{3}{5} = \frac{77}{135}$$

<u>**Bays law**</u>

Suppose that $A_1, A_2, \dots, A_n$ are mutually expulsive events such that

$$A_1 \cup A_2 \cup \dots \cup A_n = S$$

And $p(A_i) > 0 , i = 1{:}n$ then for any event B we have

$$p(B) = p(B \backslash A_1) * p(A_1) + p(B \backslash A_2) * p(A_2) + \cdots + p(B \backslash A_n) * p(A_n)$$

Proof

$$B = B \cap S$$

$$B = B \cap (A_1 \cup A_2 \cup \dots \cup A_n)$$

$$B = (B \cap A_1) \cup (B \cap A_2) \cup \dots \cup (B \cap A_n)$$

$$p(B) = p(B \cap A_1) + p(B \cap A_2) + \cdots + p(B \cap A_n)$$

$$p(B \backslash A) = \frac{p(B \cap A)}{p(A)} \;, \therefore p(B \cap A) = p(B \backslash A) * p(A)$$

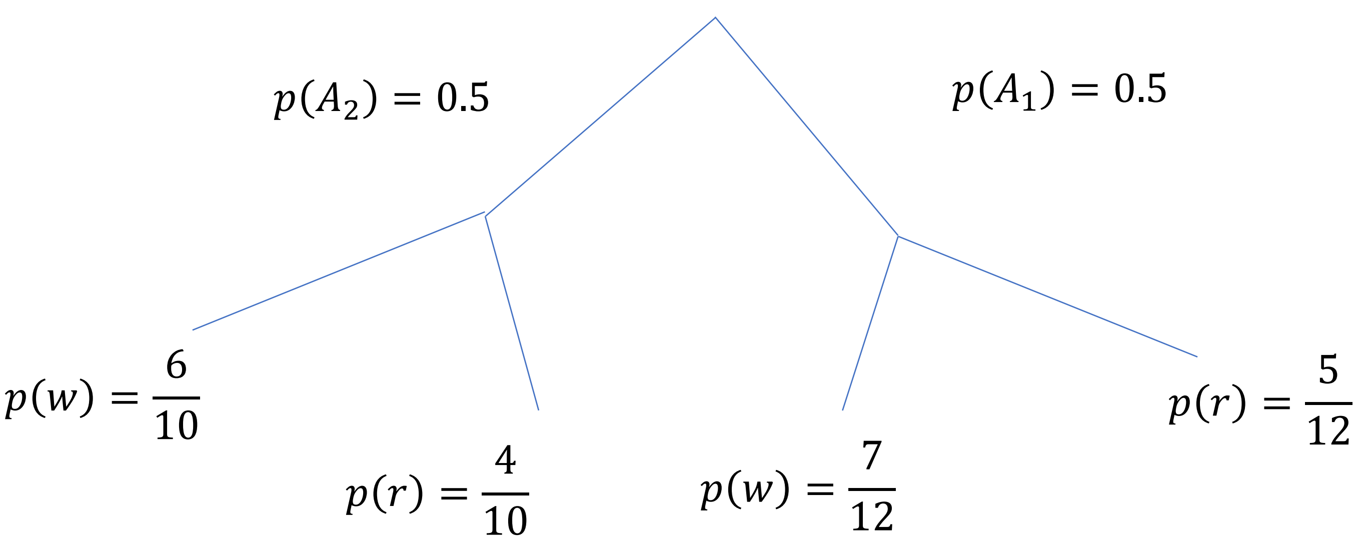
$$\therefore p(B) = p(B \backslash A_1) * p(A_1) + p(B \backslash A_2) * p(A_2) + \cdots + p(B \backslash A_n) * p(A_n)$$

Ex:consider two urns, urn A_1 consists 5 red ball and 7 white ball urn A_2 consists 6 white balls and 4 red balls one of urn is selected at random find probability that the ball will be white

sol\:$p(w) = p(w \backslash A_1)p(A_1) + p(w \backslash A_2)p(A_2)$

$$= \frac{\binom{7}{1}}{\binom{12}{1}} * \frac{1}{2} + \frac{\binom{6}{1}}{\binom{10}{1}} * \frac{1}{2} = \frac{7}{12} * \frac{1}{2} + \frac{6}{10} * \frac{1}{2} = \frac{71}{120}$$

or



Ex: in a certain College 4% of man and 1% of women are taller than 180centimeters furthermore 60% of a student are a woman if a student is selected at random

1. what is the probability that the student taller than 180 cm.?
2. give me the back of the stewarding is done around an 180 cm. What is the probability that the student is a woman?

Sol\

1. $p(t) = p(t\backslash w) * p(w) + p(t\backslash m) * p(m)$
$$= 0.01 * 0.60 + 0.04 * 0.4$$
$$= 0.006 + 0.016 = 0.022$$
2. $p(w\backslash t) = \frac{p(t\backslash w)*p(w)}{p(t)} = \frac{0.01*0.60}{0.022} = \frac{6}{22}$

Independence

Def: events A and B are independent if $p(A \cap B) = p(A) * p(B)$ otherwise they are independent

Ex:

let a Fair coin be tossed three times consider the events

A= {first toss is a head}

B= {second toss is ahead}

C= {exactly two head in a row}

Sol\

S= {TTT, HTT, THT, TTH, HHT, HTH, THH, HHH}

A= {HTT, HHT, HTH, HHH}

B= {THT, HHT, THH, HHH}

C= {HHT, THH}

$$p(A) = \frac{1}{2}, p(B) = \frac{1}{2}, p(C) = \frac{1}{4}$$

$$(A \cap B) = \{\text{HHT}, \text{HHH}\}$$

$$p(A \cap B) = \frac{1}{4}$$

$$(A \cap C) = \{\text{HHT}\}$$

$$p(A \cap C) = \frac{1}{8}$$

$$(B \cap C) = \{\text{HHT}, \text{THH}\}$$

$$p(B \cap C) = \frac{1}{4}$$

$$p(A) * p(B) = \frac{1}{4} = p(A \cap B)$$

$$\therefore A \text{ and } B \text{ are independent}$$

$$p(A) * p(C) = \frac{1}{8} = p(A \cap C)$$

$$\therefore A \text{ and } C \text{ are independent}$$

$$p(B) * p(C) = \frac{1}{8} \neq p(B \cap C)$$

$$\therefore A \text{ and } C \text{ are dependent}$$

- three events A, B and C are independent if

1. $p(A \cap B) = p(A) * p(B)$, $p(A \cap C) = p(A) * p(C)$, $p(B \cap C) = p(B) * p(C)$
2. $p(A \cap B \cap C) = p(A) * p(B) * p(C)$