

Ex: If the probability that a man will live another ten years is $\frac{1}{4}$ and the probability that his wife will live another ten years is $\frac{1}{3}$, Find the probability that

1. the two will live another ten years?
2. one of them live at least another ten years?
3. the wife lives ten years?

Sol\

$A = \{\text{man will live another ten years}\}$, $B = \{\text{wife will live another ten}\}$

$$1. p(A \cap B) = p(A)p(B) = \frac{1}{4} * \frac{1}{3} = \frac{1}{12}$$

$$2. p(A \cup B) = p(A) + p(B) - p(A \cap B) = \frac{1}{4} + \frac{1}{3} - \frac{1}{12} = \frac{1}{2}$$

$$3. p(A^c \cap B) = p(A^c)p(B) = \frac{3}{4} * \frac{1}{3} = \frac{1}{4}$$

Ex: If the probability of three men hitting a target is respectively $\frac{1}{6}$ and $\frac{1}{4}$, $\frac{1}{3}$, then if all of them aim once at the target, create the probability

1. that only one of them will hit the target?
2. If the target hits only one man, what is the probability that this man will be the first man?

Sol\

$A = \{\text{The first man hits the target}\}$, $B = \{\text{The second man hits the target}\}$,
 $C = \{\text{The third man hits the target}\}$, $E = \{\text{Suppose event E is that one man has hit the target}\}$

$$1. E = (A \cap B^c \cap C^c) \cup (A^c \cap B \cap C^c) \cup (A^c \cap B^c \cap C)$$

$$p(E) = p(A \cap B^c \cap C^c) + p(A^c \cap B \cap C^c) + p(A^c \cap B^c \cap C)$$

$$p(E) = p(A)p(B^c)p(C^c) + p(A^c)p(B)p(C^c) + p(A^c)p(B^c)p(C)$$

$$p(E) = \frac{1}{6} * \frac{3}{4} * \frac{2}{3} + \frac{5}{6} * \frac{1}{4} * \frac{2}{3} + \frac{5}{6} * \frac{3}{4} * \frac{1}{3} = \frac{1}{12} + \frac{5}{36} + \frac{5}{24} = \frac{31}{72}$$

2. Finding $p(A|E)$ is the probability that the first man hit the target knowing that only one person hit the target and the event $A \cap E = (A \cap B^c \cap C^c)$ is the first man hit the target

$$p(A|E) = \frac{p(A \cap E)}{p(E)} = \frac{\frac{1}{12}}{\frac{31}{72}} = \frac{6}{31}$$

Ex: One type of Rocket hits the target with a probability of 0.3 What is the number of Rockets that must be fired in order for the probability of hitting the target 80%?

Sol\ The probability that the Rocket misses the target is 0.7

So, the probability that the n Rocket will miss the target is 0.7^n So we're looking for the smallest integer of n where it is

$$1 - 0.7^n > 0.8 \text{ or } 0.7^n < 0.2$$

0.7	$0.7^2 = 0.49$	$0.7^3 = 0.343$	$0.7^4 = 0.2401$	$0.7^5 = 0.16804$
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Therefore, at least 5 Rocket must be fired

Ex: Three shooters shoot at the same target, each of them shoots just once. The first one hits the target with a probability of 70%, the second one with a probability of 80% and the third one with a probability of 90%. What is the probability that the shooters will hit the target?

1. at least once?
2. at least twice?

Sol\

1. $p(A) = p\{\text{at least one hit the target}\}$

$$A = \{(h \cap n \cap n) \cup (n \cap h \cap n) \cup (n \cap n \cap h) \cup (h \cap h \cap n) \\ \cup (h \cap n \cap h) \cup (n \cap h \cap h) \cup (h \cap h \cap h)\}$$

$$p(A) = p(h \cap n \cap n) + p(n \cap h \cap n) + p(n \cap n \cap h) + p(h \cap h \cap n) \\ + p(h \cap n \cap h) + p(n \cap h \cap h) + p(h \cap h \cap h)$$

$$p(A) = p(h)p(n)p(n) + p(n)p(h)p(n) + p(n)p(n)p(h) \\ + p(h)p(h)p(n) + p(h)p(n)p(h) + p(n)p(h)p(h) \\ + p(h)p(h)p(h)$$

$$= 0.7 * 0.2 * 0.1 + 0.3 * 0.8 * 0.1 + 0.3 * 0.2 * 0.9 + 0.7 * 0.8 * 0.1 \\ + 0.7 * 0.2 * 0.9 + 0.3 * 0.8 * 0.9 + 0.7 * 0.8 * 0.9$$

$$= 0.0140 + 0.024 + 0.054 + 0.056 + 0.126 + 0.216 + 0.504 \\ = 0.994$$

Or

$$p(A) = p(\text{at least one hit the target}) = 1 - p(\text{no one hit}) \\ = 1 - p(n \cap n \cap n) = 1 - p(n)p(n)p(n) \\ = 1 - 0.3 * 0.2 * 0.1 = 1 - 0.006 = 0.994$$

2. $p(B) = p\{\text{at least twice hit the target}\}$

$$B = (h \cap h \cap n) \cup (h \cap n \cap h) \cup (n \cap h \cap h) \cup (h \cap h \cap h)$$

$$p(B) = p(h \cap h \cap n) + p(h \cap n \cap h) + p(n \cap h \cap h) + p(h \cap h \cap h) \\ p(B) = p(h)p(h)p(n) + p(h)p(n)p(h) + p(n)p(h)p(h) + p(h)p(h)p(h)$$

$$p(B) = 0.7 * 0.8 * 0.1 + 0.7 * 0.2 * 0.9 + 0.3 * 0.8 * 0.9 + 0.7 * 0.8 * 0.9 \\ = 0.056 + 0.126 + 0.216 + 0.504 = 0.902$$

Ex: In the class of 30 students, seven of them don't have done the homework. The teacher chose randomly 6 students. What is the chance that at least four of them have done their homework?

Sol\ A= {at least four of them have done their homework}

$$\begin{aligned} p(A) &= \frac{\binom{23}{4}\binom{7}{2} + \binom{23}{5}\binom{7}{1} + \binom{23}{6}\binom{7}{0}}{\binom{30}{6}} \\ &= \frac{8855 * 21 + 33649 * 7 + 100947 * 1}{593775} \\ &= \frac{185955 + 235543 + 100947}{593775} \\ &= \frac{522445}{593775} = 0.8799 \end{aligned}$$

Ex:

