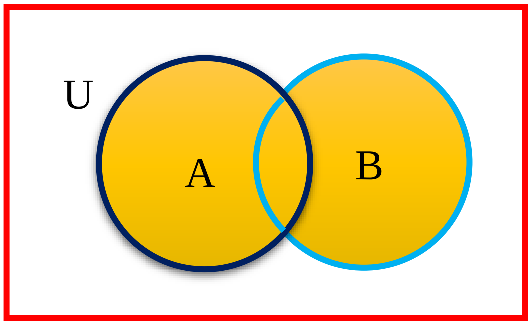
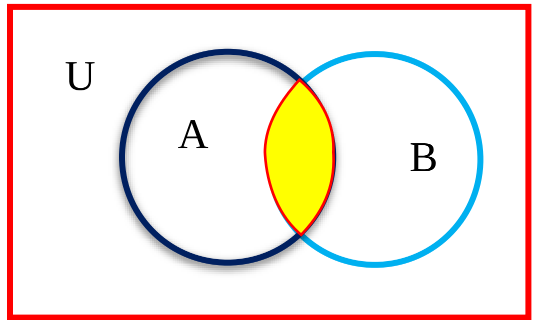
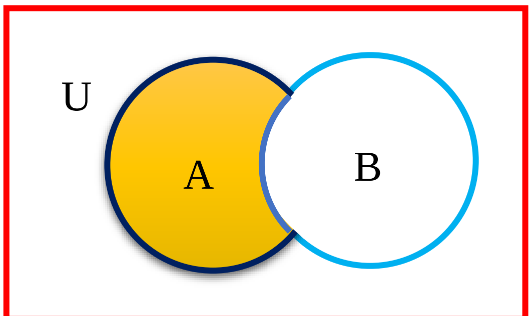


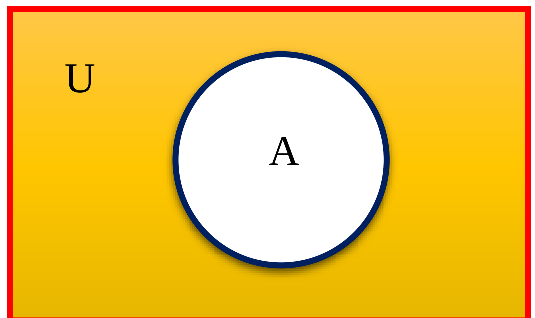

$A \cup B$




$A \cap B$




$A \backslash B$




$A^c$

6. Algebra of Sets

| | | |
|---|---|---|
| 1 | $A \cup A = A$<br>$A \cap A = A$ | Idempotent laws |
| 2 | $(A \cup B) \cup C = A \cup (B \cup C)$<br>$(A \cap B) \cap C = A \cap (B \cap C)$ | Associative laws |
| 3 | $A \cup B = B \cup A$<br>$A \cap B = B \cap A$ | Commutative laws |
| 4 | $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$<br>$A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | Distributive laws |
| 5 | $A \cup \emptyset = A$<br>$A \cap U = A$<br>$A \cup U = U$<br>$A \cap \emptyset = \emptyset$ | Identity laws |
| 6 | $(A^c)^c = A$ | Involution laws |
| 7 | $A \cup A^c = U$<br>$A \cap A^c = \emptyset$<br>$U^c = \emptyset$<br>$\emptyset^c = U$ | Complement laws |
| 8 | $(A \cup B)^c = A^C \cap B^C$<br>$(A \cap B)^c = A^C \cup B^C$ | De Morgan's laws |

Prove that $(A \cup B)^c = A^C \cap B^C$

We may prove

1. $(A \cup B)^c \subset (A^C \cap B^C)$
2. $(A^C \cap B^C) \subset (A \cup B)^c$

1. Let x $\in (A \cup B)^c$

$\therefore x \notin (A \cup B)$ definition of Complement

$\therefore x \notin A \; and \; x \notin B$ def. of union

$\therefore x \in A^C and \;\; x \in B^C$ def. of Complement

$\therefore x \in (A^C \cap B^C)$ def. of intersection

$\boxed{\therefore (A \cup B)^c \subset (A^C \cap B^C)}$ def. of subset

2. Let x $\in A^C \cap B^C$

$\therefore x \in A^C and \ \ x \in B^C$ def. of intersection

$\therefore x \notin A \ and \ x \notin B$ def. of Complement

$\therefore x \notin (A \cup B)$ def. of union

$\therefore x \in (A \cup B)^c$ def. of Complement

$\boxed{\therefore (A^C \cap B^C) \subset (A \cup B)^c}$ def. of subset

From 1 and 2 we have $(A \cup B)^c = A^C \cap B^C$

<u>Theorem:</u> if A and B are disjoint finite set then $A \cup B$ is finite and

$n(A \cup B) = n(A) + n(B)$

If a set A is finite, we let $n(A)$ denote the number of elements of A

<u>Theorem:</u> if A and B are finite set then $A \cup B$ and $A \cap B$ are finite and

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

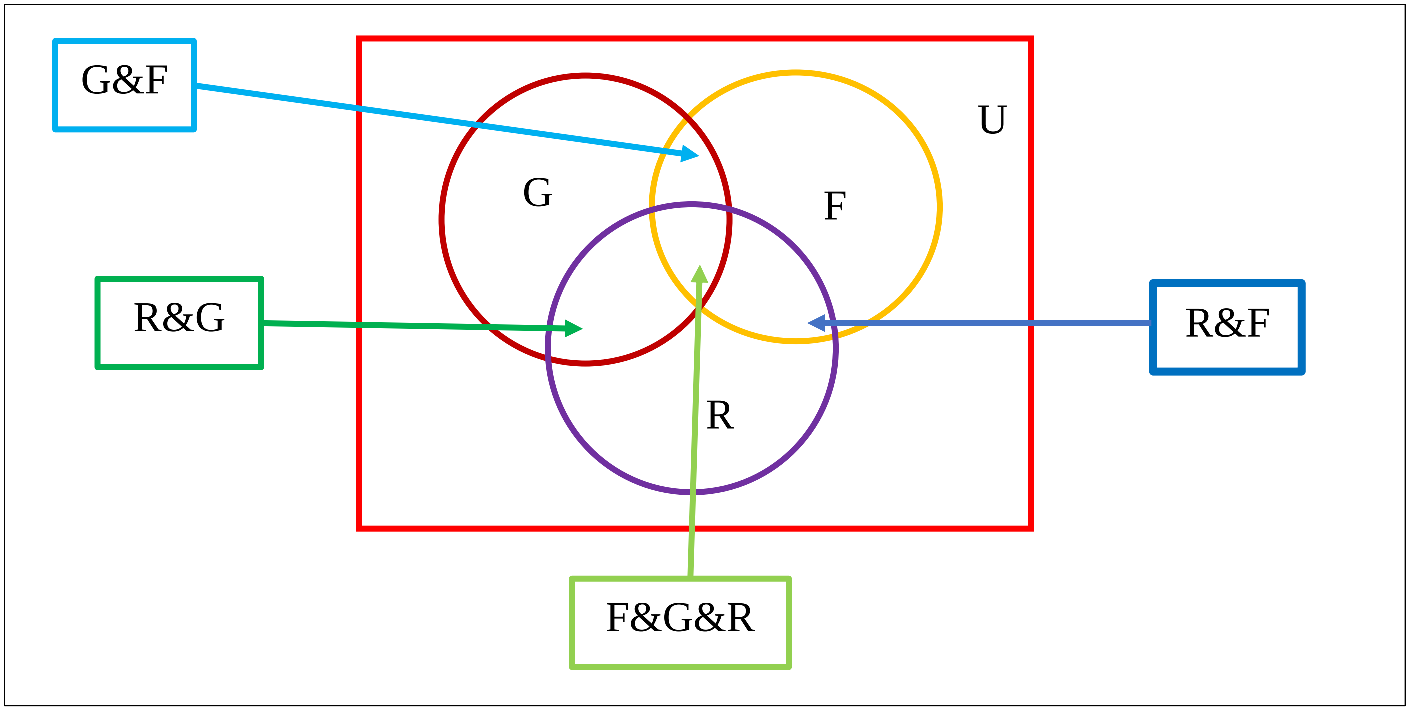
<u>Theorem:</u> if A, B and C are finite set's then

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C)$$
$$-n(B \cap C) + n(A \cap B \cap C)$$

<u>Ex</u>: suppose that 100 of 120 students at a college take at least one of the languages French, German and Russian also suppose 65 study French ,45 study German, 42 study Russian, 20 study French and German, 25 study French and Russian and 15 study German and Russian.

Find the number of students who study all three languages and to fill in correct number of students in each of the region.

Sol: let G, F, R denoted the set of students studying French, German, Russian respectively then Venn Diagram shownAs in the following figure




F=65, R=42, G=45, F&R=25, F&G=20, G&R=15

$$n(F \cup G \cup R) = n(F) + n(G) + n(R) - n(F \cap G) - n(F \cap R)$$
$$-n(G \cap R) + n(F \cap G \cap R)$$

$100 = 65 + 45 + 42 - 20 - 25 - 15 + n(F \cap G \cap R)$

$n(F \cap G \cap R) = 8$ students study three languages

20-8=12 students study French &German only

25-8=17 students study French &Russian only

15-8=7 students study Russian &German only

65-12-8-17=28 students study French only

45-12-7-8=18 students study German only

42-17-7-8=10 students study Russian only

120-100=20 students did not study any languages.

7. Duality

Suppose E is an equation of set algebra the dual E* of E is the equation obtained by replacing each occurrence of $\cup, \cap, \emptyset, U$ in E by $\cap, \cup, U, \emptyset$ respectively.

Ex: write the duality of $(A \cup B) \cap (A \cup B^c) = A \cup \emptyset$

Sol: $(A \cap B) \cup (A \cap B^c) = A \cap U$

8. Class of sets, power set
   Class of sets means that set of sets. if A is a set with n element then the set of all subset of A called power set and denoted by p(A).

Ex: suppose A= {1,2,3,4}, let B the class of subset of A which contain exactly three elements of A find B.

Sol: B= [{1,2,3}, {1,2,4}, {2,3,4}, {1,3,4}]

- if A is finite then the number of elements in p(A) is
  n p(A)=$2^{n(A)}$

Ex: suppose A= {1,2,3} find power set

Sol: n p(A)=$2^{n(A)} = 2^3 = 8$
P(A)= $[\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{A\}]$