CH 2 counting techniques

Permutation:

In mathematics, a permutation of a set, it is in general, an arrangement of its members into a sequence or linear order, or if the set is already ordered, a rearrangement of its elements, and Permutations are used in almost every branch of mathematics, and in many other fields of science.

There are two type of permutation

1. **Permutation with repetition**

These are the easiest one to calculate, when you have n things to choose from… You have n chooses each time

When choosing r of them, the Permutation are:

n*n*n*(r times)

in other word there are n possibilities for the first choice, and there are n possibilities for the second choice, etc.

$$n * n * n \ldots (r\ times) = n^r$$

Ex: suppose an urn contain 10 balls find number of ordered samples of size 3 with repetition?

Sol\

10*10*10 (3 times)=$10^3 = 1000$ permutations

Where n is the number of things choose from and you choose r of them.

- In general, if a_i objects of i^{th} kind ,i= 1,2,3,…,r, are there then the number of permutations of all $a_1 + a_2 + a_3 + \ldots + a_r$ objects are given by

$$\frac{(a_1 + a_2 + \cdots + a_r)!}{a_1!\, a_2! \ldots a_r!}$$

Theorem suppose we have n item where $n_1, n_2, \ldots, n_k$ that are identical the number of ways to permute is

$$\frac{n!}{n_1!\, n_2! \ldots n_k!}$$

And $\sum n_i = n$

$$n! = \begin{cases} n(n-1)! & if\ n > 0 \\ 1 & if\ n = 0 \end{cases}$$

- $n! = n(n-1)(n-2)(n-3) \ldots 3 * 2 * 1$
- $0! = 1$
  $3! = 3 * 2 * 1 = 6$
  $5! = 5 * 4 * 3 * 2 * 1 = 120$
  $10! = 10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1 = 3628800$

Ex: How many ways to order the letters of MISSISSIPPI?

MISSISSIPPI

Sol \ there are 11 letters, but 4 I,4 S and 2 P

there are

$$\frac{11!}{4!\,4!\,2!} = \frac{11 * 10 * 9 * 8 * 7 * 6 * 5 * 4 * 3 * 2 * 1}{4 * 3 * 2 * 1 * 4 * 3 * 2 * 1 * 2 * 1} = 34650$$

Ex: How many ways can someone choose three cards from the cards if repetition is allowed?

Sol\ $52 * 52 * 52 = 52^3 = 140608$

Ex: How many different signals each consisting of 6 Flags hang in a vertical line can be formed from 2 identical Red flag and 4 identical blue flags?

Sol\

$$\frac{6!}{4! * 2!} = \frac{6 * 5 * 4!}{4! * 2 * 1} = 15$$

rrbbbb-brrbbb-bbrrbb-bbbrrb-bbbbrr-rbrbbb-rbbrbb-rbbbrb-rbbbbr-brbbbr-bbrbbr-bbbrbr-bbbbrr-brbrbb-bbrbrb

2. Permutation without repetition

The number of Permutations of size r when chosen from a set of size n (obviously with $n \geq r$) denoted by

$$p(n, r) = \frac{n!}{(n-r)!}$$

- Sampling with repetition n^r
- Sampling without repetition

$$\frac{n!}{(n-r)!}$$

Ex: Suppose an urn Contains 8 balls find the number of order sample of size 3

1. with repetition
2. without repetition

sol\

1. $8^3 = 512$
2. $\frac{8!}{(8-3)!} = 336$

Ex: find n if

1. P(n,2) =72
2. 2p (n, 2) +50=p (2n, 2)

Sol \

1. $p(n, 2) = 72$

$$\frac{n!}{(n-2)!} = 72$$

$$\frac{n(n-1)(n-2)!}{(n-2)!} = 72$$
$$n(n-1) = 72 \Rightarrow n^2 - n - 72 = 0 \Rightarrow (n-9)(n+8) = 0$$
$$n - 9 = 0 \Rightarrow n = 9$$
$$n + 8 = 0 \Rightarrow n = -8 \quad Ignores$$

2. $2p(n,2) + 50 = p(2n,2)$

$$2\frac{n!}{(n-2)!} + 50 = \frac{2n!}{(2n-2)!}$$
$$2\left[\frac{n(n-1)(n-2)!}{(n-2)!}\right] + 50 = \frac{2n(2n-1)(2n-2)!}{(2n-2)!}$$

$$2[n(n-1)] + 50 = 2n(2n-1)$$

$$2n^2 - 2n + 50 = 4n^2 - 2n \Rightarrow 2n^2 + 50 = 4n^2$$
$$\Rightarrow 2n^2 + 50 - 4n^2 = 0$$
$$-2n^2 + 50 = 0 \Rightarrow 50 = 2n^2 \Rightarrow n^2 = 50 \Rightarrow n = \pm 5$$
$$n = 5$$

Ex: Assuming repetition is not allowed,

1. how many three-digit numbers can be arranged from the following numbers 2,3,5,6,7,9
2. How many of them are less 400?
3. How many of them are even number?
4. How many of them are odd number?
5. How many of them are a multiple of 5?

Sol \

1.

$\boxed{6}$ $\boxed{5}$ $\boxed{4}$

6*5*4=120

2.

5*4*2=40

$\boxed{2}$ $\boxed{5}$ $\boxed{4}$

3. 5*4*2=40 $\boxed{5}$ $\boxed{4}$ $\boxed{2}$

4. $\boxed{5}$ $\boxed{4}$ $\boxed{4}$
   5*4*4=80

5. 5*4*1=20 $\boxed{5}$ $\boxed{4}$ $\boxed{1}$

Ex: in how many ways can a party of 6 person arrange them selves

1. In a row of six chair
2. Around a round table

Sol \

1. $6! = 720$
2. $(n-1)! = 5! = 120$

One person can sit anywhere on the round table and the other five people can arrange themselves around the table.