CH 3 Introduction to probability

Def:

Process whose outcome are uncertain is called experiment.

Def:

Random experiment in particular random experiment is a process by which we observe something uncertain.

Def:

the set that contains all possible outcomes from the random experiment is called sample space of the experiment the sample space is denoted by S.

Def:

two events A_1 and A_2 such that $A_1 \cap A_2 = \emptyset$ are said to be mutually exclusive events

Axioms of probability

1. for every event A $0 \leq p(A) \leq 1$.
2. $p(S) = 1$.
3. If A and B are mutually exclusive events then $p(A \cup B) = p(A) + p(B)$
4. If $A_1, A_2, \dots, A_n$ is sequence of mutually exclusive events then
$$p(A_1 \cup A_2 \cup \dots \cup A_n) = p(A_1) + p(A_2) + \cdots + p(A_n)$$

Theorem:

if $\emptyset$ is empty set then $p(\emptyset) = 0$

proof:

$A \cup \emptyset = A$

$p(A \cup \emptyset) = p(A)$

$p(A) + p(\emptyset) = p(A) \Rightarrow p(\emptyset) = p(A) - p(A)$

$p(\emptyset) = 0$

Theorem:

$p(A^c) = 1 - p(A)$

proof:

$A \cup A^c = S$

$p(A \cup A^c) = p(S)$

$p(A) + p(A^c) = 1 \Rightarrow p(A^c) = 1 - p(A)$

$p(A) = 1 - p(A^c)$

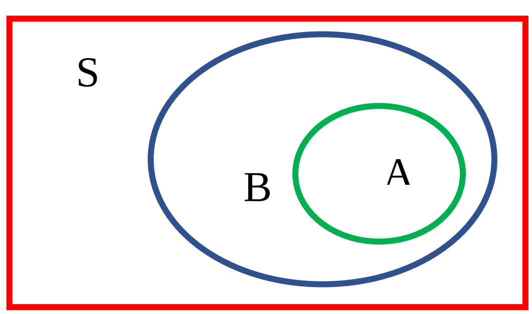
Theorem:

if A⊂B then $p(A) \leq p(B)$

proof:

$B = A \cup (B \backslash A)$

$p(B) = p(A) + p(B \backslash A)$



$but\ p(B \backslash A) \geq 0$

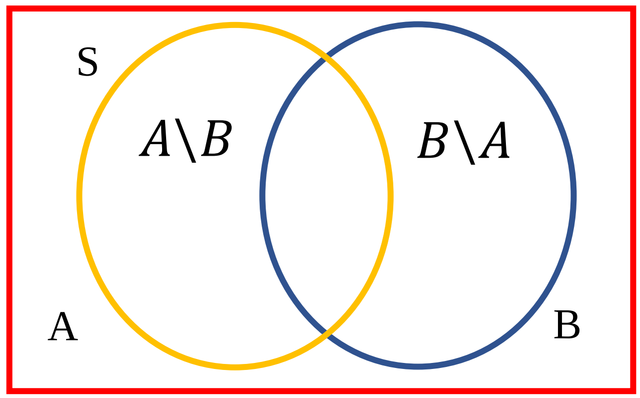
$\therefore p(A) \leq p(B)$

<u>Theorem:</u>

If A and B are two events then

$p(A \backslash B) = p(A) - p(A \cap B)$

<u>proof:</u>



$A = (A \backslash B) \cup (A \cap B)$

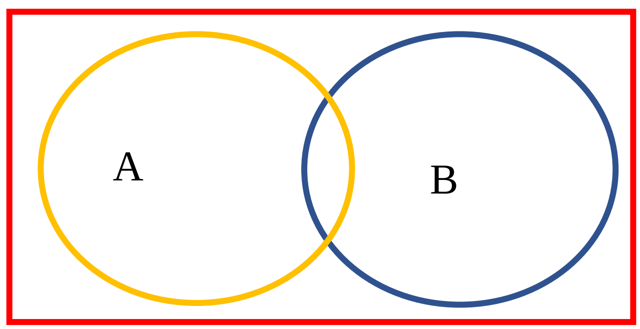
$p(A) = p(A \backslash B) + p(A \cap B)$

$\therefore p(A \backslash B) = p(A) - p(A \cap B)$

<u>Theorem:</u>

If A and B are two events then $p(A \cup B) = p(A) + p(B) - p(A \cap B)$

<u>proof:</u>



$A \cup B = (A \backslash B) \cup B$

$p(A \cup B) = p(A \backslash B) + p(B)$

$p(A \cup B) = p(A) - p(A \cap B) + p(B)$

$\therefore p(A \cup B) = p(A) + p(B) - p(A \cap B)$

- $p(A \cup B \cup C) = p(A) + p(B) + p(C) - p(A \cap B) - p(A \cap C) - p(B \cap C) + p(A \cap B \cap C)$

<u>Ex:</u>

let three coins tossed and the number of head observed then find

1. The probability that at least one head appears?

Sol\

n(S)= $2^3 = 8$

S= {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

A= {at least one head appears}

$$p(A) = p(one\ head) + p(two\ head) + p(three\ head)$$

$$p(0) = \frac{1}{8}, p(1) = \frac{3}{8}, p(2) = \frac{3}{8}, p(3) = \frac{1}{8}$$

$$p(A) = p(1) + p(2) + p(3) = \frac{3}{8} + \frac{3}{8} + \frac{1}{8} = \frac{7}{8}$$

Or

$$p(\text{at least one head appears}) = 1 - p(no\ head)$$

$$= 1 - \frac{1}{8} = \frac{7}{8}$$