Ex: The probability of mutually exclusive events A and B are related as $p(B) = \{p(A)\}^2$ and $A \cup B = S$.

Find $p(A)$ and show that $p(A) = p(B^c)$.

Sol\ let $p(A) = p$

$$A \cup B = S$$

$$p(A \cup B) = p(S) \Longrightarrow p(A) + p(B) = p(S)$$

$$p + p^2 = 1 \Longrightarrow p^2 + p - 1 = 0$$

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1 - 4(1)(-1)}}{2(1)} = \frac{-1 \pm \sqrt{5}}{2}$$

$$\therefore t_1 = \frac{-1 + \sqrt{5}}{2}, t_2 = \frac{-1 - \sqrt{5}}{2}$$

- We neglect t_2???

$$\therefore p(A) = t_1 = \frac{-1 + \sqrt{5}}{2}$$

$$p(B) = \{p(A)\}^2 \Longrightarrow \left(\frac{-1 + \sqrt{5}}{2}\right)^2$$

$$p(B) = \frac{\left(-1 + \sqrt{5}\right)^2}{4} = \frac{\left(1 - 2\sqrt{5} + 5\right)}{4} = \frac{6 - 2\sqrt{5}}{4} = \frac{2\left(3 - \sqrt{5}\right)}{4}$$

$$p(B) = \frac{3 - \sqrt{5}}{2}$$

$$p(B^c) = 1 - p(B) = 1 - \frac{3 - \sqrt{5}}{2} = \frac{2 - \left(3 - \sqrt{5}\right)}{2} = \frac{2 - 3 + \sqrt{5}}{2}$$

$$p(B^c) = \frac{-1 + \sqrt{5}}{2}$$

$\therefore\ p(A) = p(B^c)$

Ex: 3 horses A, B and C are a race A twice as likely to win as B and B is twice as likely to win as C what are their respective the probability of win?

Sol\ let the $prob.C\ winc = p$

$\therefore\ prob.B\ winc = 2p$

$\therefore\ prob.A\ winc = 4p$

$$p + 2p + 4p = 1$$

$$7p = 1 \Longrightarrow p = \frac{1}{7}$$

$$\therefore p(C) = \frac{1}{7}\ , p(B) = \frac{2}{7}\ , p(A) = \frac{4}{7}$$

Ex: Let a cared be selected at random from playing cards

1. What is the prob. that the card is spade?
2. What is the prob. that the card is face?
3. What is the prob. that the card is spade &face?

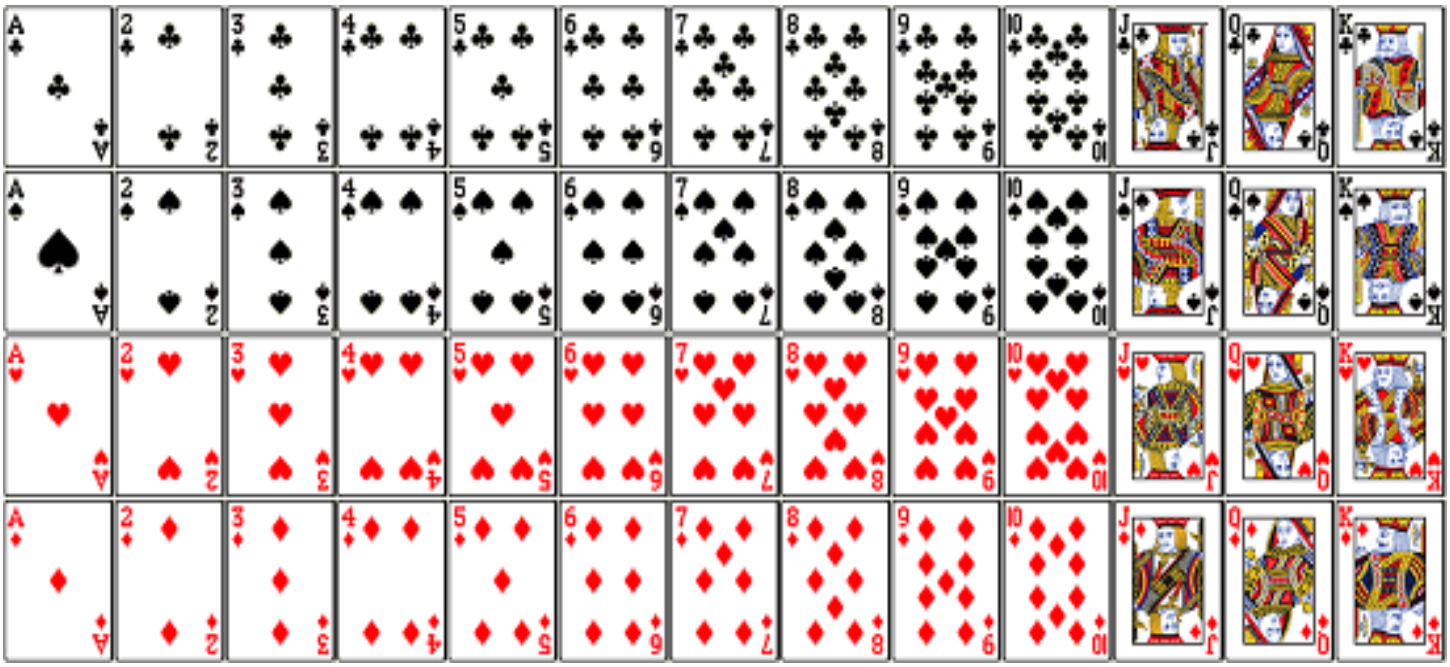
name	shape
Spades	
Diamonds	
Hearts	
Clubs	

Sol\

$$1.\ p(cards\ is\ spades) = \frac{\binom{13}{1}}{\binom{52}{1}} = \frac{13}{52}$$

$$2.\ p(cards\ is\ face) = \frac{\binom{12}{1}}{\binom{52}{1}} = \frac{12}{52}$$

$$3.\ p(cards\ is\ spade\ \&face) = \frac{\binom{3}{1}}{\binom{52}{1}} = \frac{3}{52}$$



Ex: two cards be selected at random from playing cards find the probability that

1. Both two cards are Spades?
2. One of them is Spades and the other one is Hearts?

Sol\

1. Let A= Both two cards are Spades

$$p(A) = \frac{\binom{13}{2}}{\binom{52}{2}} = \frac{78}{1326}$$

2. Let B= One of them is Spades and the other one is Hearts

$$p(B) = \frac{\binom{13}{1}\binom{13}{1}}{\binom{52}{2}} = \frac{169}{1326}$$

Ex: Three light bulbs were randomly selected from 15 light bulbs, 5 of which were defective. Find the probability

1. all of them are non-defective?
2. Just one defective?
3. at least one defective?

Sol\

1. let A = (all bulbs non defective)

$$p(A) = \frac{\binom{10}{3}}{\binom{15}{3}} = \frac{120}{455}$$

2. let A = (Just one bulb defective)

$$p(A) = \frac{\binom{5}{1}\binom{10}{2}}{\binom{15}{3}} = \frac{5 * 45}{455} = \frac{225}{455}$$

3. let A = (at least one defective)

$$p(A) = 1 - p(\text{all bulbs non defective})$$
$$= 1 - \frac{120}{455} = \frac{67}{91}$$