

## Ch4

### Conditional probability and independence

#### Conditional probability

In probability theory, conditional probability is a measure of the probability of an event occurring, given that another event (by assumption) has already occurred. If the event of interest is A and the event B is known or assumed to have occurred, "the conditional probability of A given B", or "the probability of A under the condition B", is usually written as  $P(A|B)$ , or sometimes  $P_B(A)$  or  $P(A/B)$ . For example, the probability that any given person has a cough on any given day may be only 5%. But if we know or assume that the person is sick, then they are much more likely to be coughing.

Given two events A and B then conditional probability know as follows

$$p(A | B) = \frac{p(A \cap B)}{p(B)}, p(B) > 0$$

$$P(A \cap B) = P(A | B)P(B)$$

Ex: Two digits are selected at random from the digits from (1 to 9) if the sum is even find probability that both numbers are even

Sol\ The sum is even if both numbers are odd or if both numbers are even

There are 5 odd numbers (1,3,5,7,9) there are  $\binom{5}{2} = 10$  ways to choose tow odd numbers  $\{(1,3), (1,5), (1,7), (1,9), (3,5), (3,7), (3,9), (5,7), (5,9), (7,9)\}$

There are 4 even numbers (2,4,6,8) there is  $\binom{4}{2} = 6$  ways to choose two even numbers  $\{(2,4), (2,6), (2,8), (4,6), (4,8), (6,8)\}$

There are  $6 + 10 = 16$  ways to choose to number such that there's some is even

Let A be the event that both numbers even  $= \binom{4}{2} = 6$

Let B be the event that the sum is even  $= \binom{4}{2} + \binom{5}{2} = 16$

$$p(A \setminus B) = \frac{p(A \cap B)}{p(B)} = \frac{\frac{\binom{4}{2}}{\binom{9}{2}}}{\frac{\binom{4}{2} + \binom{5}{2}}{\binom{9}{2}}} = \frac{6}{16}$$

Ex: Let a pair of fair dice is tossed. If the sum is 6, find the probability that one of the two dice is the number 2

Sol\

$E = \{\text{sum} = 6\}$

$A = \{\text{one dice is } 2\}$

$E = \{(1,5), (5,1), (2,4), (4,2), (3,3)\}$

$A = \{(2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (1,2), (3,2), (4,2), (5,2), (6,2)\}$

$$p(A \cap E) = \frac{2}{36}, p(E) = \frac{5}{36}$$

$$p(A \setminus E) = \frac{\frac{2}{36}}{\frac{5}{36}} = 2/5$$

Ex:

In a certain College 25% of students failed math 15% failed chemistry and 10% of students failed both math and chemistry. A student selected at random

1. if he failed math what is the probability that he failed chemistry
2. if he failed chemistry what is the probability that he failed math

3. what is the probability that he failed chemistry or math

sol\

$$p(\text{math}) = 0.25, p(\text{chem}) = 0.15, p(\text{math} \cap \text{chem}) = 0.10$$

$$1. p(\text{ch} \setminus \text{math}) = \frac{p(\text{math} \cap \text{chem})}{p(\text{math})} = \frac{0.10}{0.25} = \frac{2}{5}$$

$$2. p(\text{math} \setminus \text{ch}) = \frac{p(\text{math} \cap \text{chem})}{p(\text{ch})} = \frac{0.10}{0.15} = \frac{2}{3}$$

$$3. p(\text{math} \cup \text{chem}) = p(\text{math}) + p(\text{chem}) - p(\text{math} \cap \text{chem}) = 0.15 + 0.25 - 0.10 = 0.30$$

Ex: a given Population of 1550 student are classified according to their gender into two group (70% male and 30% female) suppose that 80% from each gender are passes a certain examination (success) the remain do not pass (fail) at student is selected at random what is the probability that the selected student is meal provided that he success

Sol\

$$\text{Number of mails} = 1550 * 0.7 = 1085$$

$$\text{Number of females} = 1550 * 0.3 = 465$$

	success	fail	total
Mail	868	217	1085
female	372	93	465
	1240	310	1550

$$p(m \setminus s) = \frac{\frac{868}{1550}}{\frac{1240}{1550}} = \frac{868}{1240}$$

## Multiplication. Theorem for conditional probability

Theorem:

$$p(E \cap A) = p(E) * P(A|E)$$

- For any event  $A_1, A_2, \dots, A_n$

$$p(A_1 \cap A_2 \cap \dots \cap A_n) =$$

$$p(A_1) * p(A_2|A_1) * p(A_3|A_1 \cap A_2) * \dots * p(A_n|A_1 \cap A_2 \cap \dots \cap A_{n-1})$$

Ex:

a lot contains 12 items which 4 are defective 3 items are drawn at random from a lot one after the another find the probability that all 3 items are non-defective.

Sol\

$$\begin{aligned} p(\text{all 3 item are non - defective}) &= \frac{\binom{8}{1}}{\binom{12}{1}} * \frac{\binom{7}{1}}{\binom{11}{1}} * \frac{\binom{6}{1}}{\binom{10}{1}} \\ &= \frac{8 * 7 * 6}{12 * 11 * 10} = \frac{14}{55} \end{aligned}$$