

Def:

Random experiment is an experiment having the following properties:

1. The outcome can not be predicate with certainly.
2. The outcome can describe prior to the performance.
3. It can be repeated under the same conditions.

For examples

- Tossing a coin.
- Choice of a real number between 0 and 1.
- Drawing a card from playing card.

Distribution function of discreet r.v.

Distribution function of discreet r.v. is called probability mass function (p.m.f) and satisfies the conditions

- $0 \leq p(x_i) \leq 1$.
- $\sum_{i=1}^{\infty} p(x_i) = 1$.

Ex: let X be a discreet r.v. with the following p.m.f.

$$p(X = x) = \begin{cases} 0.1 & x = 0.2 \\ 0.2 & x = 0.4 \\ 0.2 & x = 0.5 \\ 0.3 & x = 0.8 \\ 0.2 & x = 1 \\ 0 & o.w \end{cases}$$

Find

1. $p(X \leq 0.5)$
2. $p(0.25 \leq X \leq 0.75)$
3. $p(X = 0.2 | X < 0.6)$

Sol

1. The event $X \leq 0.5$ can happen only if X is 0.2,0.4,0.5

$$\begin{aligned}
 p(X \leq 0.5) &= p(X = 0.2) + p(X = 0.4) + p(X = 0.5) \\
 &= 0.1 + 0.2 + 0.2 = 0.5 \\
 2. \quad p(0.25 \leq X \leq 0.75) &= p(X = 0.4) + p(X = 0.5) \\
 &= 0.2 + 0.2 = 0.4 \\
 3.
 \end{aligned}$$

$$\begin{aligned}
 p(X = 0.2 \mid X < 0.6) &= \frac{p(X = 0.2) \text{ and } p(X < 0.6)}{p(X < 0.6)} \\
 &= \frac{0.1}{0.1 + 0.2 + 0.2} = 0.2
 \end{aligned}$$

Ex: suppose the probability mass function of r.v. X is

x	-2	-1	0	1	2
$p(x)$	$\frac{c}{10}$	$\frac{4c}{10}$	0.1	0.2	0.2

Find

1. The value of c.
2. $p(-1 < X \leq 2)$

Sol\

$$1. \sum_{i=1}^{\infty} p(x_i) = 1.$$

$$\begin{aligned}
 \frac{c}{10} + \frac{4c}{10} + 0.1 + 0.2 + 0.2 &= 1 \\
 \frac{5c}{10} + 0.5 &= 1 \Rightarrow \frac{5c}{10} + \frac{5}{10} = 1 \\
 \frac{5c + 5}{10} &= 1 \Rightarrow 5c + 5 = 10 \\
 5c &= 5 \Rightarrow c = 1
 \end{aligned}$$

x	-2	-1	0	1	2
$p(x)$	0.1	0.4	0.1	0.2	0.2

$$2. p(-1 < X \leq 2) = 0.1 + 0.2 + 0.2 = 0.5$$